LONGITUDINAL DISPLACEMENT OF PLASTIC MASS BETWEEN NON-CIRCULAR CYLINDERS

(PRODOL'NOE PEREMESHCHENIYE PLASTICHESKOI MASSY MEZHDY NEKRUGOVYMI TSILINDRAMI)

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The present paper deals with longitudinal displacement of mass between rough non-circular cylinders. A method is developed which permits a reduction of solution for various problems with non-linear law of deformation of a certain form to a solution of the same problems employing linear law. The longitudinal displacement between cylinders is considered when contours of cross-sections are confocal ellipses.

1. Fundamental relationships. Let us investigate the longitudinal displacement of a plastic mass between rough cylinders. Assume that the inner cylinder is displaced by an amount ω in the negative direction of the z-axis and the outer cylinder remains stationary.

Obviously, the components of displacement $u = v = 0$, and the stress and strain components will be

$$\sigma_x = \sigma_y = \sigma_z = \sigma_0, \qquad \tau_{xy} = 0, \qquad \varepsilon_x = \varepsilon_y = \varepsilon_z = \gamma_{xy} = 0.$$

The remaining stress components $\tau_{zx} = \tau_x$, $\tau_{yx} = \tau_y$ and strain components $\gamma_{zx} = \gamma_x$, $\gamma_{yz} = \gamma_y$, and also the displacement components w, do not depend upon z. They are functions of x, y only.

For this condition the differential equations of equilibrium are particularly simple. They are reduced to a single equation

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = 0 \tag{1.1}$$

The strain components γ_x and γ_y are expressed in terms of w in the following manner

$$2\gamma_x = \frac{\partial w}{\partial x}, \qquad 2\gamma_y = \frac{\partial w}{\partial y} \tag{1.2}$$

The fundamental relationships between stress and strain components have the usual form

$$\gamma_x = \frac{\gamma}{\tau}\tau_x, \qquad \gamma_y = \frac{\gamma}{\tau}\tau_y \tag{1.3}$$

while

$$\tau^2 = \tau_x^2 + \tau_y^2, \qquad \gamma^2 = \gamma_x^2 + \gamma_y^2$$

The condition of plasticity is determined in a certain form of relationships between τ and γ, namely

$$\tau = \tau(\gamma), \quad \text{or} \quad \gamma = \gamma(\tau) \tag{1.4}$$

On the basis of (1.1), the stress components τ_x and τ_y may be expressed as a function of ψ as follows

$$\tau_x = \frac{\partial\psi}{\partial y}, \qquad \tau_y = -\frac{\partial\psi}{\partial x} \tag{1.5}$$

and following (1.2) the strain components γ_x and γ_y may be represented as a function of ϕ

$$2k\gamma_x = \frac{\partial\varphi}{\partial x}, \qquad 2k\gamma_y = \frac{\partial\varphi}{\partial y}, \qquad \varphi = k(w + W) \tag{1.6}$$

where k is a mechanical constant which will be introduced later.

Fundamental relationships (1.3), together with (1.5) and (1.6), provide a system of equations

$$\frac{\partial\varphi}{\partial x} = 2k\frac{\gamma}{\tau}\tau_x, \qquad \frac{\partial\varphi}{\partial y} = 2k\frac{\gamma}{\tau}\tau_y, \qquad \frac{\partial\psi}{\partial x} = -\tau_y, \qquad \frac{\partial\psi}{\partial y} = \tau_x \tag{1.7}$$

Now examine the conditions existing along the contours of the bounding cylinders. Consider that adhesion exists along these contours. Then, obviously, on the contour of the inner movable cylinder there will be

$$w = -W, \quad \text{or} \quad \varphi = 0$$

and on the contour of the outer stationary cylinder there will be

$$w = 0, \quad \text{or} \quad \varphi = kW$$

Now let us calculate the friction force Q which acts on the inner cylinder from the side of the plastic mass.

Since the component τ_n, which acts along the contour of the cross-section of the inner cylinder, appears in the form

$$\tau_n = \tau_x \cos(n, x) + \tau_y \cos(n, y) = \tau_x \frac{dy}{ds} - \tau_y \frac{dx}{ds} = \frac{d\psi}{ds}$$

it follows that the friction force per unit length of the cylinder is equal to

$$Q = \oint \tau_n ds = \oint d\psi \tag{1.8}$$

System of equation (1.7) becomes simplest when τ and γ are connected by linear relationships

$$\tau = 2k\gamma, \quad \text{or} \quad \gamma = \frac{\tau}{2k} \tag{1.9}$$

which determine the usual elastic state and contain a single mechanical constant k.

As will be shown later, the system of equations (1.7) also can be reduced to quite a convenient form when τ and γ are nonlinearly related

$$\tau = \frac{2k\gamma}{\sqrt{1+(2m\gamma)^2}}, \quad \text{or} \quad \gamma = \frac{\tau/2k}{\sqrt{1-(m\tau/k)^2}} \tag{1.10}$$

describing a plastic state with hardening and containing two mechanical constants k and m.

Notice that if w is assumed to be a longitudinal velocity and γ_x and γ_y are assumed to indicate strain rates, then the derived equations describe the longitudinal flow of plastic mass between rough cylinders. Naturally, in such a case, the mechanical constants k and m and also W assume new dimensions.

2. Transformation of equations. Now consider the system of basic equations

$$\frac{\partial\varphi}{\partial x} = \frac{\tau_x}{\sqrt{1-(m\tau/k)^2}}, \quad \frac{\partial\varphi}{\partial y} = \frac{\tau_y}{\sqrt{1-(m\tau/k)^2}}, \quad \frac{\partial\psi}{\partial x} = -\tau_y, \quad \frac{\partial\psi}{\partial y} = \tau_x \tag{2.1}$$

which corresponds to system (1.7), given previously.

Express τ_x and τ_y in terms of the modulus τ of the shear stress vector and the angle of inclination θ of this vector to the x-axis, such that

$$\tau_x = \tau\cos\theta, \qquad \tau_y = \tau\sin\theta \tag{2.2}$$

Transform system of equations (2.1) as follows

$$\begin{aligned} \frac{\partial\varphi}{\partial x} &= \frac{\tau\cos\theta}{\sqrt{1-(m\tau/k)^2}} \qquad & \frac{\partial\varphi}{\partial y} &= \frac{\tau\sin\theta}{\sqrt{1-(m\tau/k)^2}} \\ \frac{\partial\psi}{\partial x} &= -\tau\sin\theta, & \frac{\partial\psi}{\partial y} &= \tau\cos\theta \end{aligned} \tag{2.3}$$

and introduce a new quantity t by means of equalities

$$t = \frac{2\tau}{1+\sqrt{1-(m\tau/k)^2}}, \qquad \tau = \frac{t}{1+(mt/2k)^2} \qquad (0 \leqslant t \leqslant 2k/m) \tag{2.4}$$

For convenience we will use dimensionless quantities

$$x' = \frac{x}{l}, \qquad y' = \frac{y}{l}, \qquad \varphi' = \frac{\varphi}{kW}, \qquad \psi' = \frac{\psi}{kW}$$

and also

$$t' = \frac{lt}{kW}, \qquad \tau' = \frac{l\tau}{kW}, \qquad w' = \frac{w}{W}, \qquad \mu = \frac{mW}{2l}$$

where l is some characteristic length.

For brevity let us agree to omit the primes. That is, let us denote the dimensionless quantities by the same symbols as the dimensional quantities.

Substituting t for τ, and passing over to dimensionless quantities, let us rewrite the system of equations (2.3) in the following form

$$\begin{aligned} \frac{\partial\varphi}{\partial x} &= \frac{t\cos\theta}{1-\mu^2t^2}, \qquad & \frac{\partial\varphi}{\partial y} &= \frac{t\sin\theta}{1-\mu^2t^2} \\ -\frac{\partial\psi}{\partial x} &= \frac{t\sin\theta}{1+\mu^2t^2}, \qquad & \frac{\partial\psi}{\partial y} &= \frac{t\cos\theta}{1+\mu^2t^2} \end{aligned} \tag{2.5}$$

and the equalities (2.4) as

$$t = \frac{2\tau}{1+\sqrt{1-(2\mu\tau)^2}}, \qquad \tau = \frac{t}{1+\mu^2t^2} \tag{2.6}$$

Note that for $\mu = 0$ equations (2.5) will reduce the usual equations valid for a linear law (1.0), and equalities (2.6) indicate that $t = \tau$.

Let us perform a substitution of variables using the transformation formulas

$$\frac{\partial\varphi}{\partial x} = \frac{1}{\Delta}\frac{\partial y}{\partial\psi}, \qquad \frac{\partial\varphi}{\partial y} = -\frac{1}{\Delta}\frac{\partial x}{\partial\psi}, \qquad \frac{\partial\psi}{\partial x} = -\frac{1}{\Delta}\frac{\partial y}{\partial\varphi}, \qquad \frac{\partial\psi}{\partial y} = \frac{1}{\Delta}\frac{\partial x}{\partial\varphi}$$

assuming ϕ and ψ to be the independent variables, and x and y to be the sought functions.

Finally, system of equations (2.5) will be transformed into the system

$$\begin{aligned} \frac{\partial x}{\partial\varphi} &= \frac{1-\mu^2t^2}{t}\cos\theta, \qquad & \frac{\partial y}{\partial\varphi} &= \frac{1-\mu^2t^2}{t}\sin\theta, \\ -\frac{\partial x}{\partial\psi} &= \frac{1+\mu^2t^2}{t}\sin\theta, \qquad & \frac{\partial y}{\partial\psi} &= \frac{1+\mu^2t^2}{t}\cos\theta \end{aligned} \tag{2.7}$$

and the determinant of the transformation Δ will be

$$\Delta = \frac{\partial x}{\partial\varphi}\frac{\partial y}{\partial\psi} - \frac{\partial x}{\partial\psi}\frac{\partial y}{\partial\varphi} = \frac{1}{t^2} - \mu^4t^2 \tag{2.8}$$

Let us introduce complex quantities

$$z = x + iy, \qquad \omega = \varphi + i\psi, \qquad T = \ln\frac{1}{t} + i\theta$$

and the corresponding conjugate quantities which will be identified by dashes.

Multiplying the second and the fourth equations of system (2.7) by i and then adding them to the first and the third equations respectively, we obtain

$$\frac{\partial z}{\partial \varphi} = \frac{1 - \mu^2 t^2}{t} e^{i\theta}, \qquad \frac{\partial z}{\partial \psi} = i \frac{1 + \mu^2 t^2}{t} e^{i\theta} \tag{2.9}$$

Substitution of quantities w, w for ϕ and ψ and substitution of T, $\overline{T}$ for t and θ will result in

$$\frac{\partial z}{\partial \omega} = e^T, \qquad \frac{\partial z}{\partial \overline{\omega}} = -\mu^2 e^{-\overline{T}} \tag{2.10}$$

From this it immediately follows that

$$dz = e^T d\omega - \mu^2 e^{-\overline{T}} d\overline{\omega}, \qquad \frac{\partial T}{\partial \overline{\omega}} = \mu^2 t^2 \frac{\partial \overline{T}}{\partial \omega} \tag{2.11}$$

Since the complex quantities $\partial T/\partial \overline{\omega}$ and $\partial \overline{T}/\partial \omega$ are conjugate, and since the variable quantity μt is real and varies within the limits $0 < \mu t < 1$, then $\partial T/\partial \overline{\omega} = 0$. Therefore, the complex quantity T is an arbitrary analytic function of the complex variable ω only, namely,

$$\frac{e^{i\theta}}{t} = e^T, \qquad T = T(\omega) \tag{2.12}$$

Note that for the linear law (1.9), or for $\mu = 0$, equations (2.11) will be of the form

$$dz = e^T d\omega \tag{2.13}$$

and equations (2.12) become

$$\frac{e^{i\theta}}{\tau} = e^T, \qquad T = T(\omega) \tag{2.14}$$

Along with the complex variable z, it is convenient to introduce an auxiliary complex variable ζ as follows:

$$\zeta = \zeta(\omega), \qquad d\zeta = e^T d\omega$$

Whereby, in performing this, the stress field in the z-plane will correspond to a certain auxiliary stress field in the ζ-plane which occurs for the linear law (1.9).

It is easy to observe that equation (2.11) can be transformed to the form

$$dz = \frac{d\zeta}{d\omega} d\omega - \mu^2 \frac{d\overline{\omega}}{d\overline{\zeta}} d\overline{\omega} = d\zeta - \mu^2 \left(\frac{d\overline{\omega}}{d\overline{\zeta}}\right)^2 d\overline{\zeta} \tag{2.15}$$

and equation (2.12) can be rewritten as

$$\frac{e^{i\theta}}{t} = \frac{d\zeta}{d\omega} \tag{2.16}$$

The above equations provide an opportunity to find a solution of the present problem of a longitudinal displacement of a plastic mass between rough cylinders, when a solution of the same problem for the linear law (1.9) is known.

In fact, knowing the function $\zeta = \zeta(\omega)$, it is not difficult by way of integration of equations (2.15) to find the function

$$z = z(\omega, \bar{\omega})$$

The region in the z-plane, which is occupied by a plastic mass between the curvilinear contours, will have the same form as the region in the ζ-plane. However, the forms of curvilinear contours which bound the indicated regions will differ somewhat. The diameters of these contours are determined by the parameters which enter into the solution, and which may be assigned previously.

3. Confocal ellipses. Consider for example a field of shear stresses and longitudinal displacements when the inner and outer contours of the cross-sections of the cylinders are confocal ellipses:

$$\frac{x^2}{\mathrm{ch}^2\alpha} + \frac{y^2}{\mathrm{sh}^2\alpha} = l^2, \qquad \frac{x^2}{\mathrm{ch}^2\beta} + \frac{y^2}{\mathrm{sh}^2\beta} = l^2$$

shown in Fig. 1.

Again as before, let us employ dimensionless quantities, taking as the characteristic length l the distance between the center O and the focus L.

First consider the linear law (1.9). That is, assume that the parameter $\mu = 0$. The solution of the formulated problem is expressed by a function

$$z = \mathrm{ch}\,\Omega, \qquad \Omega = (\beta - \alpha)\,\omega + \alpha$$

Determining the derivative $dz/d\omega = (\beta - \alpha)\,\mathrm{sh}\,\Omega$, we obtain

$$\frac{e^{i\theta}}{\tau} = (\beta - \alpha)\,\mathrm{sh}\,\Omega$$

Now assume the nonlinear law (1.10), that is, assume the parameter $\mu \neq 0$. The solution of the problem is then given by the function

$$\zeta = c\,\mathrm{ch}\,\Omega, \quad \Omega = (b - a)\,\omega + a$$

which contains arbitrary parameters a, b and c.

Euqations (2.15) after introduction of ζ, can be transformed into the following form without much effort

$$dz = c \operatorname{sh} \Omega \, d\Omega - \frac{\mu^2}{c\,(b-a)^2} \frac{d\bar{\Omega}}{\operatorname{sh} \bar{\Omega}} = d\zeta - \frac{\mu^2}{(b-a)^2} \frac{d\bar{\zeta}}{\bar{\zeta}^2 - c^2}$$

Integrating these equations and making a final selection of arbitrary constants, we find

$$z = c \operatorname{ch} \Omega + \frac{\mu^2}{c\,(b-a)^2} \tanh^{-1} \frac{1}{\operatorname{ch} \bar{\Omega}} = \zeta + \frac{\mu^2}{c\,(b-a)^2} \tanh^{-1} \frac{c}{\bar{\zeta}} \tag{3.1}$$

As a result of substitution of ζ, equation (2.16) can be rewritten as

$$\frac{e^{i\theta}}{t} = c\,(b-a) \operatorname{sh} \Omega \tag{3.2}$$

In equations (3.1) and (3.2) let us separate the real and imaginary parts. For convenience introduce the notations

$$\Phi = (b-a)\,\varphi + a, \qquad \Psi = (b-a)\,\psi$$

Coordinates x and y are determined as functions of Φ and Ψ in the form

$$\frac{x}{c} = \operatorname{ch} \Phi \cos \Psi + \frac{\mu^2}{c^2\,(b-a)^2} \tanh^{-1} \frac{\cos \Psi}{\operatorname{ch} \Phi}$$

$$\frac{y}{c} = \operatorname{sh} \Phi \sin \Psi + \frac{\mu^2}{c^2\,(b-a)^2} \tan^{-1} \frac{\sin \Psi}{\operatorname{sh} \Phi} \tag{3.3}$$

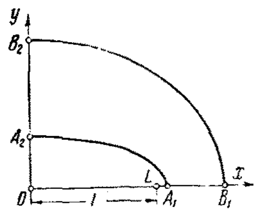

Fig. 1.

and quantities t and θ are again expressed as functions of Φ and Ψ as

$$\frac{1}{t} = c\,(b-a) \sqrt{\operatorname{ch}^2 \Phi - \cos^2 \Psi}, \qquad \operatorname{tg} \theta = \frac{\operatorname{tg} \Psi}{\operatorname{th} \Phi} \tag{3.4}$$

Lines of equal displacements and lines of action of shear stresses in the xy-plane may be constructed on the basis of (3.3) by assuming $\Phi =$ const and $\Psi =$ const respectively.

The contour of the moving inner cylinder is the line along which $\phi = 0$. The coordinates x and y for this contour, as well as quantities t and θ, can be obtained from (3.3) and (3.4) for $\phi = 0$ or $\Phi = a$.

Equations for the inner contour are determined in the following manner

$$\frac{x}{c} = \operatorname{ch} a \cos \Psi + \frac{\mu^2}{c^2\,(b-a)^2} \tanh^{-1} \frac{\cos \Psi}{\operatorname{ch} a}$$

$$\frac{y}{c} = \operatorname{sh} a \sin \Psi + \frac{\mu^2}{c^2\,(b-a)^2} \tan^{-1} \frac{\sin \Psi}{\operatorname{sh} a} \tag{3.5}$$

and quantities t and θ along this inner contour will be

$$\frac{1}{t} = c\,(b-a) \sqrt{\operatorname{ch}^2 a - \cos^2 \Psi}, \qquad \operatorname{tg} \theta = \frac{\operatorname{tg} \Psi}{\operatorname{th} a} \tag{3.6}$$

The contour of the outer stationary cylinder is the line along which $\phi = 1$. The coordinates x and y for this contour, as well as quantities t and θ, may be obtained from (3.3) and (3.4) for $\phi = 1$ or $\Phi = b$.

Equations for the outer contour are expressed as

$$\frac{x}{c} = \operatorname{ch} b \cos \Psi + \frac{\mu^2}{c^2 (b-a)^2} \tanh^{-1} \frac{\cos \Psi}{\operatorname{ch} b}$$
$$\frac{y}{c} = \operatorname{sh} b \sin \Psi + \frac{\mu^2}{c^2 (b-a)^2} \tan^{-1} \frac{\sin \Psi}{\operatorname{sh} b} \tag{3.7}$$

and quantities t and θ along this outer contour will be

$$\frac{1}{t} = c(b-a)\sqrt{\operatorname{ch}^2 b - \cos^2 \Psi}, \qquad \operatorname{tg} \theta = \frac{\operatorname{tg} \Psi}{\operatorname{th} b} \tag{3.8}$$

The friction force Q per unit length of the cylinder is easily found from (1.8). Since Ψ acquires an increment of 2π when integrated along the inner contour, then

$$\oint d\Psi = 2\pi \quad \text{and} \quad Q = 2\pi \frac{kW}{b-a}$$

In these solutions the parameters a, b and c are related to each other by the two equations

$$\operatorname{ch} a + \frac{\mu^2}{c^2 (b-a)^2} \tanh^{-1} \frac{1}{\operatorname{ch} a} = \frac{\operatorname{ch} \alpha}{c}, \quad \operatorname{sh} a + \frac{\mu^2}{c^2 (b-a)^2} \tan^{-1} \frac{1}{\operatorname{sh} a} = \frac{\operatorname{sh} \alpha}{c} \tag{3.9}$$

which follow from equations (3.5) and from requirements that the inner contour should pass through points $x = \operatorname{ch} \alpha$, $y = 0$ and $x = 0$, $y = \operatorname{sh} \alpha$.

Besides this, parameters a, b and c are connected by two more equations

$$\operatorname{ch} b + \frac{\mu^2}{c^2 (b-a)^2} \tanh^{-1} \frac{1}{\operatorname{ch} b} = \frac{\operatorname{ch} \beta}{c}, \quad \operatorname{sh} b + \frac{\mu^2}{c^2 (b-a)^2} \tan^{-1} \frac{1}{\operatorname{sh} b} = \frac{\operatorname{sh} \beta}{c} \tag{3.10}$$

which follow from equations (3.7) and from requirements that the inner contour should pass through points $x = \operatorname{ch} \beta$, $y = 0$ and $x = 0$, $y = \operatorname{sh} \beta$.

Three parameters a, b and c are determined from three equations of system (3.9) and (3.10), while the fourth equation is not satisfied. In this way the outer and inner contours are symmetrical about axes x and y. They pass only through assigned points A_1, A_2 and B_1, while the fourth point B_2 remains aside.

It is especially interesting to find out the variation of shear stress along x and y axes. From (3.3) and (3.4) for $\Psi = 0$ the coordinate x and the quantity t along the x-axis will be determined as follows

$$\frac{x}{c} = \operatorname{ch} \Phi + \frac{\mu^2}{c^2 (b-a)^2} \tanh^{-1} \frac{1}{\operatorname{ch} \Phi}, \qquad \frac{1}{t} = c(b-a) \operatorname{sh} \Phi \tag{3.11}$$

and the coordinate y and the quantity t along the y-axis are found from (3.3) and (3.4) for $\Psi = 1/2\pi$ and will be

$$\frac{y}{c} = \operatorname{sh}\Phi + \frac{\mu^2}{c^2(b-a)^2}\tan^{-1}\frac{1}{\operatorname{sh}\Phi}, \qquad \frac{1}{t} = c(b-a)\operatorname{ch}\Phi \tag{3.12}$$

One should bear in mind that the longitudinal displacement w is expressed in terms of Φ in the simple form

$$w = -\frac{b-\Phi}{b-a}$$

Note, that within the considered region of Φ and Ψ variation, that is, for $a < \Phi < b$, $0 \leqslant \Psi \leqslant 2\pi$, the determinant of transformation Δ must be different from zero. Otherwise Φ and Ψ will not be single-valued functions of x and y. Since the determinant is

$$\Delta = \frac{1}{t^2} - \mu^4 t^2 > 0, \qquad \text{or} \qquad \frac{1}{t} > \mu$$

then from (3.3) it follows that

$$c(b-a)\sqrt{\operatorname{ch}^2\Phi - \cos^2\Psi} > \mu$$

This condition will be satisfied if parameters a, b and c are subjected to the following limitation

Notice that for $\mu = 0$, equations (3.9) and (3.10) will give

$$a = \alpha, \quad b = \beta, \quad c = 1$$

Let us carry out a numerical example for

$$\mu = 0.2, \qquad \alpha = 0.4, \qquad \beta = 1.0$$

which would illustrate the preceding discussion. Parameters a, b and c are obtained from solution of equations (3.9) and (3.10). They are equal to

$$a = 0.347, \qquad b = 1.105, \quad c = 0.889\ .$$

The coordinates x and y for points of inner contour are determined from (3.5), and the corresponding values of t and τ at these points are found from (3.6). In this manner

$\Psi = 0.000$	0.314	0.628	0.942	1.257	1.571
$x = 1.081$	1.011	0.841	0.603	0.315	0.000
$y = 0.000$	0.153	0.265	0.345	0.394	0.411
$t = 4.194$	3.159	2.163	1.680	1.462	1.399
$\tau = 2.462$	2.258	1.822	1.510	1.347	1.297

The coordinates x and y for points of outer contour are determined from (3.7) and the corresponding values of t and τ are found from (3.8). Hence,

$\Psi = 0.000$	0.314	0.628	0.942	1.257	1.571
$x = 1.543$	1.467	1.246	0.904	0.475	0.000
$y = 0.000$	0.387	0.734	1.009	1.184	1.245
$t = 1.104$	1.076	1.012	0.946	0.901	0.886
$\tau = 1.053$	1.028	0.972	0.913	0.873	0.859

These results make it possible to conjecture the contour forms for cross-sections of bounding cylinders and the character of variation in shear stress τ along inner and outer contours.

The values of t, τ and w along the x-axis are determined from (3.11). Their final values are equal to

$\Phi = 0.347$	0.498	0.650	0.801	0.953	1.105
$x = 1.081$	1.112	1.174	1.266	1.388	1.543
$t = 4.194$	2.858	2.130	1.667	1.344	1.104
$\tau = 2.462$	2.154	1.803	1.500	1.253	1.053
$-w = 1.000$	0.800	0.600	0.400	0.200	0.000

Analogous values of t, τ and w, however, along the y-axis are found from (3.12). They are equal to

$\Phi = 0.347$	0.498	0.650	0.801	0.953	1.105
$y = 0.411$	0.547	0.695	0.857	1.039	1.245
$t = 1.399$	1.317	1.217	1.108	0.996	0.886
$\tau = 1.297$	1.231	1.149	1.056	0.958	0.859
$-w = 1.000$	0.800	0.600	0.400	0.200	0.000

These results indicate the character of variation of shear stress τ and longitudinal displacement w along x- and y-axes.

Contours of cross-sections for bounding cylinders are shown in Fig. 2. In the same figure, by dotted lines, are also shown

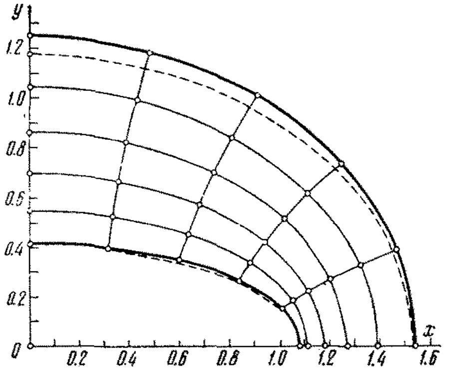

Fig. 2.

contours of cross-sections for $\mu = 0$ which are confocal ellipses. In the same figure is drawn the lattice Φ = constant and Ψ = constant, which are constructed for various values of Φ (from 0.347 to 1.105) for equal intervals of 0.152 and for various values of Ψ (from 0.000 to 1.571), also for equal intervals of 0.314.

In conclusion we note that the technique presented here makes it possible to investigate also other problems of longitudinal displacement of plastic mass between non-circular cylinders.

Translated by V.A.V.